SUPPLEMENTARY PROBLEMS FOR CHAPTER 2

1. A zero-mean random vector $\boldsymbol{x}$ has the correlation matrix

$$\mathbf{R}_{\boldsymbol{x}} = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$$

Find three distinct linear transformations that result in a random vector with uncorrelated components. What are the resulting variances of the components?

2. Tell if the following are legitimate correlation matrices. If not, tell *why* not.

(a)

$$\begin{bmatrix} 3 & 2\jmath \\ 2\jmath & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1+\jmath \\ 1-\jmath & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$$

3. Tell if the following could serve as legitimate density functions for a random vector. If not, state why.

(a)

$$f_{\boldsymbol{x}}(\mathbf{x}) = 1\ ; \qquad \mathrm{x}_1^2 + \mathrm{x}_2^2 \leq 1; \quad -1 \leq \mathrm{x}_3 \leq 1$$

(b)

$$f_{\boldsymbol{x}}(\mathbf{x}) = \frac{1}{2\pi\sqrt{3}} \exp - \mathbf{x}^{*T} \begin{bmatrix} \frac{2}{3} & -\frac{1}{3}\jmath \\ \frac{1}{3}\jmath & \frac{2}{3} \end{bmatrix} \mathbf{x}$$

(c)

$$f_{\boldsymbol{x}}(\mathbf{x}) = \frac{1}{2\pi\sqrt{3}} \exp - \mathbf{x}^{T} \begin{bmatrix} 1 & -1 \\ -1 & \frac{2}{3} \end{bmatrix} \mathbf{x}$$

4. A two-dimensional random vector has a covariance matrix given by

$$\mathbf{C}_{\boldsymbol{x}} = \begin{bmatrix} 2 & -0.3 \\ -0.3 & 2 \end{bmatrix}$$

Find three different transformations in the form $\boldsymbol{y} = \mathbf{A}\boldsymbol{x}$ that result in a random vector with uncorrelated components. For each case specify the matrix $\mathbf{A}$ and the covariance matrix of the new vector $\boldsymbol{y}$.

5. A random vector $\boldsymbol{x}$ is claimed to be characterized by the covariance matrix

$$\mathbf{C}_{\boldsymbol{x}} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

(a) Is this a legitimate covariance matrix? Prove your answer.

(b) Find two *distinct* linear transformations that would result in a random vector with *uncorrelated components*. In each case find the *covariance matrix* of the transformed vector. (Note: A transformation is not distinct if the rows of the matrix are merely permuted or reversed in sign.)

6. Tell if the following are legitimate correlation matrices and why or why not.

(a)

$$\mathbf{R}_{\boldsymbol{x}} = \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}$$

(b)

$$\mathbf{R}_{\boldsymbol{x}} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

(c)

$$\mathbf{R}_{\boldsymbol{x}} = \begin{bmatrix} 3 & \jmath \\ \jmath & 3 \end{bmatrix}$$

7. A 2-dimensional random vector $\boldsymbol{x}$ has the mean vector and covariance matrix given below:

$$\mathbf{m}_{\boldsymbol{x}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \mathbf{C}_{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   A new random vector $\boldsymbol{y}$ is defined by the linear transformation

$$\boldsymbol{y} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \boldsymbol{x}$$

   (a) What is the mean of $\boldsymbol{y}$?

   (b) What is the covariance matrix for $\boldsymbol{y}$?

8. A zero-mean random vector has the correlation matrix

$$\mathbf{R}_{\boldsymbol{x}} = \begin{bmatrix} 6 & 3 \\ 3 & 6 \end{bmatrix}$$

   (a) Find two *distinctly different* linear transformations of the form

$$\boldsymbol{y} = \mathbf{A}\boldsymbol{x}$$

   such that the components of $\boldsymbol{y}$ are uncorrelated.

   (b) What is the correlation matrix for $\boldsymbol{y}$ corresponding to each of your answers above?